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Probability of Hitting and Destroying a Surface Target for Artillery Rockets

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Artillery rockets of medium and long range are used on the battlefield for indirect firing of distant targets. Having a large dispersion of hits, they are used for targeting surface areas from multitube launchers. At the same time, the line of sight of the target does not exist, so the firing elements are taken based on the known geographical coordinates of the target and the launcher, using the ballistic computer and firing tables. Long ranges, low initial velocity and long duration of the flight to the target make such rockets very sensitive to the impact of disturbances on the accuracy of hitting. As the range increases, the accuracy of rocket systems decreases more and more. At the same time, for effective coverage of the target area, a large number of rockets is counted. Firing a target area with a large number of rockets involves a statistical approach to determining the efficiency of the firing. Thereby, statistical indicators of the distribution of impact points are used, such as the circular error probable, which represents the radius of the circle that contains half of all hits. In addition to the number of rockets with which an area is targeted, the probability of destroying all targets in the area also depends on the radius of the lethal effect of the warheads. This paper presents determination of the probability of hitting and destroying a surface target.

Key words: **artillery rockets, dispersion, probability, impact points, target.**

Introduction

ARTILLERY rockets have significant place in the modern warfare based on the numerous advantages it achieves in comparison to classic artillery: longer range, higher concentration of firepower in a short time, low cost of launchers and their high mobility. On the other hand, the price of rockets is higher than the price of classic artillery ammunition and the accuracy of rocket systems is significantly lower.

The default standard for the accuracy of artillery rockets is that the circular error probable is about 1% of the range. In the case of targets (areas) at distances less than 20 km, the dispersion of impact points ensures an even distribution of hits throughout the target area. As firing ranges increase, the number of rockets needed to cover a given area increases quadratically, which makes the use of rocket artillery economically unprofitable.

Dispersion of impact points has a normal distribution so points with a large miss become a reality with the use of a large number of rockets, which entails a high possibility of collateral damage.

The use of artillery rockets on the modern battlefield implies stricter requirements for accuracy, in order to minimize collateral damage. Also, longer ranges are required, so that the operating forces are as far as possible from the enemy. These two requirements are contradictory, because increasing the range has the effect of decreasing the accuracy. On the other hand, there is a need to produce weapons with the lowest possible price. In the case of artillery rockets, this can be achieved by increasing accuracy and ensuring target destruction with fewer rockets. The task of these rockets is to achieve a circular error probable small enough so the impact points can be considered as point hits. [1]

In order to predict the dispersion of impact points and the effect on the target, this paper presents the probability of target destruction. Predicting the possible distribution of impact points and the probability of target destruction is important in order to optimize the amount of ammunition and other resources used for a destruction of specific surface targets.

Theoretical basis

Distribution of unguided missile hits

Rockets are different from other artillery projectiles because they leave the launch tube at a low speed and after that in a relatively short flight phase, increase the speed by the action of the rocket motor. This phase is called the active phase of flight. [1]

Acceleration of the rocket under the action of the reactive force without limitation of the trajectory by the launcher causes the angular disturbances that are converted into a

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deviation of the position of the center of mass from the reference trajectory. [1] [3] [4]

The angular size of the deviation of the rocket axis from the velocity vector - angle of attack, occurs when the rocket descends from the launch device, but also during flight due to side wind.

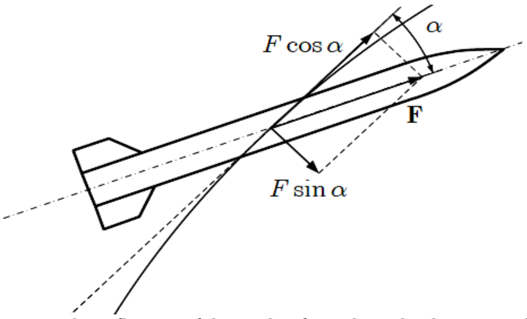

Figure 1. The influence of the angle of attack on the deviation of the rocket from the trajectory [1]

The angle at which the artillery rockets are launched is determined by ballistic calculation for the given position of the target and the launcher, as well as for the atmospheric conditions at the launch position and along the trajectory of the rocket. [1] [3] [4]

As artillery rockets are mostly fired from multitube launchers, the error of determination and positioning are constant for all rockets from a single salvo, while the error due to the oscillations of the launcher is an independent random variable.

The mean value of the error (standard deviation) occurred when positioning the firing elements and the error due to the oscillations of the launcher are conditioned by the quality of the launcher itself, while the error of determining the firing elements is conditioned by the quality of the fire control system. [1] [3] [4]

Observing a set of hits as a statistical sample, we can characterize it by mathematical expectation (desired impact point) and standard deviation as a measure of dispersion of hits. In external ballistics, instead of the standard deviation σ, as a measure of the dispersion of impact points, the size of the probable error is more often used, namely the probable error in the range V_x and the probable error in the direction V_z. [1] [3] [4]

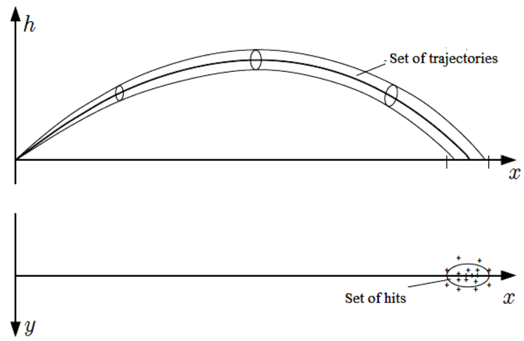

Figure 2. Dispersal of artillery projectiles [1]

Probable error is defined as:

$$\int_{-V}^{V} p(x)dx = 0,5 \tag{1}$$

where is:

p(x) - the probability density function of the random variable *x*. [1]

In the case of a normal (Gaussian) distribution, we have:

$$p(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-m)^2}{2\sigma^2}} \tag{2}$$

When firing rockets, two types of errors appear: errors of the mean hit of a group of rockets fired under the same conditions, and errors of the hit deviation from the average hit. The dispersion image of hit coordinates has the shape of an ellipse or a circle. The coordinates of the medium hit of the rocket are calculated as follows: [2]

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} \quad \bar{z} = \frac{\sum_{i=1}^{n} z_i}{n} \tag{3}$$

where *n* is the number of launched rockets.

If we look at the set of impact points, the value of the probable error represents the limits of the interval that contains 50% of the hits. The relationship between standard deviation and probable error is given by: [2]

$$\begin{aligned} V &= 0,6745\sigma \\ V_x &= 0,6745\sigma_d = 0,6745 \cdot \sqrt{\frac{\sum\left(x_i - \bar{x}\right)^2}{n-1}} \\ V_z &= 0,6745\sigma_p = 0,6745 \cdot \sqrt{\frac{\sum\left(z_i - \bar{z}\right)^2}{n-1}} \end{aligned} \tag{4}$$

For the maximum deflection $\pm 4V_x$ and $\pm 4V_z$ are taken. The radius of the circular error probable can be calculated numerically as follows: [2]

$$\begin{aligned} & V_x = V_z \\ & r_{50} = 1,1774\sigma = 1,746V \\ & \sigma = \frac{\sigma_x + \sigma_z}{2} \\ & V = \frac{V_x + V_z}{2} \end{aligned} \tag{5}$$

or:

$$\begin{aligned} & V_x \neq V_z \\ & r_{50} = 0,615\sigma_x + 0,562\sigma_z \\ & r_{50} = 0,589 \cdot (\sigma_x + \sigma_z) \end{aligned} \tag{6}$$

Probability of destroying the target

When firing a target with multiple rockets, system errors are repeated. It is assumed that firing is independent and that there is no correlation of hits. If the probability of destroying the target of individual rockets is P_{ui}, then the total probability is equal: [2]

$$P_u = 1 - \prod_{i=1}^{n}\left(1 - P_{ui}\right) \tag{7}$$

Numerical analysis of the probability of destroying a surface target for artillery rockets was done for a surface target with dimensions of 150 x 100 m. Numerical calculations were performed with the assumption that the target destruction probabilities are as defined in the following statement. The following probabilities are assumed: if total area of the target is covered by all of the warheads declared lethality radius at their impact points in amount of 30% (0.3) - it is considered disabled, and if total area of the target is covered by all of the warheads declared lethality radius at their impact points in amount of 50% (0.5) - it is considered destroyed. Covering a target by all of the warheads declared lethality radius at their impact points over 50% is considered overkill and a waste of resources.

Under the condition that probability of destruction of all individual rockets is P, probability of destroying the target with n rockets is as follows:

$$P_{un} = 1 - \left(1 - P\right)^n \tag{8}$$

When firing with a circular dispersion in the center of a surface target, probability of destruction with n rockets is as follows: [2]

$$P_{un} = 1 - e^{-\rho^2 \frac{nS_k}{\pi V_k^2}} \qquad S_k = R_u^2 \pi \tag{9}$$

where are:
ρ – constant with a value of 0.4769,
R_u – the radius of the destruction zone,
V_k - mean circular deviation,
S_k – surface of the destruction zone.

Numerical analysis

Surface target with dimensions of 150 x 100 m represents a platoon in attack and it's a standard target for rocket artillery. Numerical analysis was done in two cases. In the first case, the probability of destruction of the mentioned surface target at a range of 20 km is presented, and in the second case, at a range of 40 km.

1st case - In the first case, at a range of 20 km, the required number of rockets to destroy a surface target was calculated using equation 9 for both assumed probabilities: 30% and 50%. Lethal effect of the warhead is 60 m. Probable dispersions by distance and direction as well as standard deviations are given in Table 1.

Table 1: Probable dispersions and standard deviations by range and direction at a range of 20 km

V_x [m]	239
V_z [m]	127
σ_x [m]	354
σ_z [m]	188

If the task is to disable the target with a probability of destruction of 30% (P_{un} = 0.3), the following number of rockets is required at a range of 20 km:

$$P_{un} = 1 - e^{-\rho^2 \frac{nS_k}{\pi V_k^2}} \Rightarrow$$
$$\Rightarrow n = -\frac{V_k^2 \cdot \ln\left(1 - P_{un}\right)}{R_u^2 \cdot \rho^2} \tag{10}$$
$$n = -\frac{183^2 \cdot \ln\left(1 - 0,3\right)}{60^2 \cdot \left(0,4769\right)^2} = 14,58$$

If the task is to destroy the target with a probability of destruction of 50% (P_{un} = 0.5), the following number of rockets is required at a range of 20 km:

$$P_{un} = 1 - e^{-\rho^2 \frac{nS_k}{\pi V_k^2}} \Rightarrow$$
$$\Rightarrow n = -\frac{V_k^2 \cdot \ln\left(1 - P_{un}\right)}{R_u^2 \cdot \rho^2} \tag{11}$$
$$n = -\frac{183^2 \cdot \ln\left(1 - 0,5\right)}{60^2 \cdot \left(0,4769\right)^2} = 28,39$$

Coverage of a surface with 15 rockets and a probability of destruction of 30% target is shown in Figure 3 and the coverage of the surface with 29 rockets and a probability of destruction of 50% target is shown in Figure 4. The dispersion of impact points is based on the normal distribution of hits in relation to the defined dispersions in Table 1. [3] [4] [5]

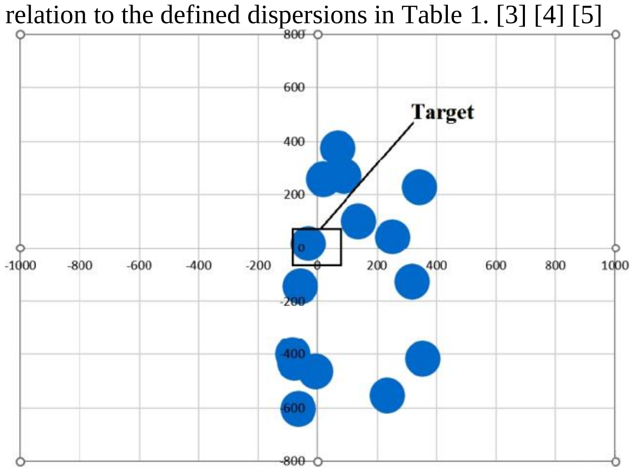

Figure 3: Dispersion of impact points with target destruction probability of 30% at range of 20 km

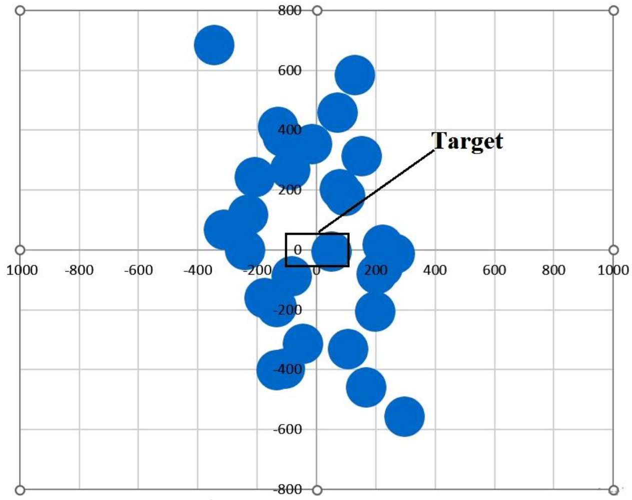

Figure 4: Dispersion of impact points with target destruction probability of 50% at a range of 20 km

2nd case - In the second case, at a range of 40 km, the required number of rockets to destroy a surface target was calculated using equation 9 for both assumed probabilities: 30% and 50%. Lethal effect of the warhead is 60 m. Probable dispersions by distance and direction as well as standard deviations are given in Table 2.

Table 2: Probable dispersions and standard deviations by range and direction at a range of 40 km

V_x [m]	411
V_z [m]	535
σ_x [m]	609
σ_z [m]	793

If the task is to disable the target with a probability of destruction of 30% (P_{un} = 0.3), the following number of rockets is required at a range of 40 km:

$$P_{un} = 1 - e^{-\rho^2 \frac{nS_k}{\pi V_k^2}} \Rightarrow$$

$$\Rightarrow n = -\frac{V_k^2 \cdot \ln\left(1 - P_{un}\right)}{R_u^2 \cdot \rho^2} \quad (12)$$

$$n = -\frac{473^2 \cdot \ln\left(1 - 0,3\right)}{60^2 \cdot \left(0,4769\right)^2} = 97,45$$

If the task is to destroy the target with a probability of destruction of 50% (P_{un} = 0.5), the following number of rockets is required at a range of 40 km:

$$P_{un} = 1 - e^{-\rho^2 \frac{nS_k}{\pi V_k^2}} \Rightarrow$$

$$\Rightarrow n = -\frac{V_k^2 \cdot \ln\left(1 - P_{un}\right)}{R_u^2 \cdot \rho^2} \quad (13)$$

$$n = -\frac{473^2 \cdot \ln\left(1 - 0,5\right)}{60^2 \cdot \left(0,4769\right)^2} = 189,41$$

Coverage of a surface with 98 rockets and a probability of destruction of 30% target is shown in Figure 5 and the coverage of the surface with 190 rockets and a probability of destruction of 50% target is shown in Figure 6. The dispersion of impacts points is based on the normal distribution of hits in relation to the defined dispersions in Table 2. [3] [4] [5]

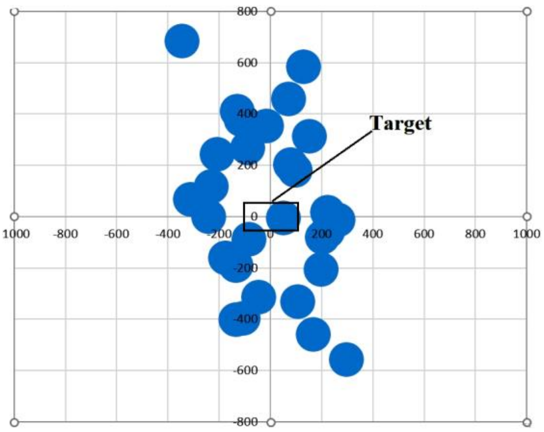

Figure 5: Dispersion of impact points with target destruction probability of 30% at a range of 40 km

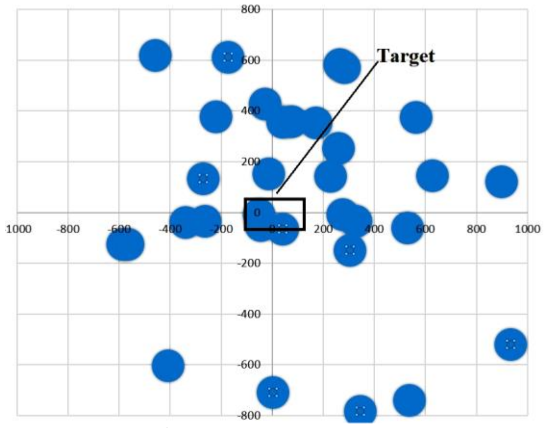

Figure 6: Dispersion of impact points with target destruction probability of 50% at a range of 40 km

The lethality circles of the warheads are shown with the assumption that the probability of destruction within the circle is 100%, while the probability of destruction outside the circle is 0%. In a practical case, the distribution of the probability of destruction in the circle and outside the circle is variable.

Conclusion

In this paper, a numerical and comparative analysis of the probability of destruction of a surface target with dimensions of 150 x 100 m by rocket artillery was performed. Numerical analysis is performed assuming that the target destruction probabilities are known and that the number of rockets required in order to neutralize the target is unknown. There were two cases at different ranges under the same conditions. For both cases, the assumption is that probable deviations and standard deviations in range and direction are known (determined by flight experiments in the field, equation 4).

In the first case, at a range of 20 km, the required number of rockets was calculated in order to achieve a probability of destroying the target of 30% and 50%. To disable a target at 30% coverage 15 rockets are required and to destroy it at 50% coverage 29 rockets are required (equations 10 and 11). The normal distribution of impact points of the required number of rockets for both probabilities is shown in Figures 3 and 4.

In the second case, at a range of 40 km, the required number of rockets was calculated in order to achieve a probability of destroying the target of 30% and 50%. To disable a target at 30% coverage 98 rockets are required and to destroy it at 50% coverage 190 rockets are required (equations 12 and 13). The normal distribution of impact points of the required number of rockets for both probabilities is shown in Figures 5 and 6.

Simulations and comparative analyzes were performed in order to optimize ammunition consumption and reduce collateral damage for the same target at a different range. As the range of artillery rockets increases, the probability of destroying the target decreases and the number of rockets required to disable the target increases exponentially.

A proposal for further research is to focus on increasing the accuracy of artillery rockets at longer ranges. One way to do this is to install special precision guidance kit (PGK) correction modules.

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Verovatnoća pogađanja i uništenja površinskog cilja za artiljerijske rakete

Artiljerijske rakete srednjeg i dugog dometa se koriste na bojnom polju za indirektno gađanje udaljenih meta. Budući da imaju veliku disperziju pogodaka, koriste se za gađanje površinskih ciljeva i lansiraju se iz višecevnih lansera raketa. U isto vreme, linija nišanjenja ne postoji i elementi za gađanje se određuju na osnovu poznatih geografskih koordinata lansera i cilja, koristeći se balističkim računarom i tablicama gađanja. Artiljerijske rakete imaju velika rasturanja pogodaka na cilju i malu preciznost gađanja iz više razloga: veliki domet, mala početna brzina rakete i dugo vreme leta rakete do cilja. Ukoliko je veći domet rakete, preciznost rakete je sve manja. Za efikasnu pokrivenost površinskog cilja potreban je veliki broj raketa. Lansiranje velikog broja raketa za sobom ostavlja pitanje efikasnosti gađanja. U skladu sa navedenim, u teoriji spoljne balistike se koriste statistički indikatori rasturanja pogodaka na cilju, kao što su CEP (verovatna kružna greška), koja predstavlja radijus kruga koji sadrži minimalno polovinu padnih tačaka od ukupnog broja lansiranih raketa. Verovatnoća uništenja površinskog cilja zavisi kako od broja lansiranih raketa, tako i od radijusa ubojnog dejstva bojeve glave. U ovom radu je predstavljeno određivanje verovatnoće pogađanja i uništenja površinskog cilja.

Ključne reči: **artiljerijske rakete, disperzija, verovatnoća, padne tačke, cilj.**