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Modeling of Shaped Charge Jet Penetration Depth: Analytical and Numerical Approach

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Shaped charge is the most effective armor-piercing mechanism, harnessing explosive charge detonation energy to form and accelerate a hypervelocity metal penetrator known as a jet. The process entails intricate dynamics including detonation wave propagation, its interaction with the metal liner, and subsequent liner collapse leading to jet formation. While both analytical and numerical models offer insight into this complex process, each approach presents distinct challenges. Analytical models, while conceptually straightforward, often rely on simplifications that compromise accuracy. Conversely, uncertainty or even unavailability of relevant material properties and high computational cost are the most important drawbacks of numerical models. Notably, the jet penetration phase imposes significantly greater computational demands compared to preceding processes of jet formation. This research aims at providing a deeper understanding of the jet interaction with target, as well as on determining its influence on penetration depth. We revisit an analytical model based on the virtual origin concept and complement it with numerical simulations using Abaqus/Explicit in a pure Eulerian domain. Through comprehensive analysis, we explore various jet parameters – such as kinetic energy, diameter, length, velocity gradient, and effective standoff distance – and their impact on penetration depth. The insights derived from this study hold practical significance for the preliminary evaluation of the shaped charge's effectiveness and consequent refinement of the design of shaped charge projectiles or warheads.

Key words: **shaped charge, jet formation, jet penetration, penetration depth, numerical simulation.**

Introduction

THE shaped charge effect is still the most effective armor-piercing mechanism. Essentially, the working principle is based on focusing or directing the energy of explosion and its conversion in formation of a hypervelocity projectile, so-called shaped charge jet, from the metal liner [1-4]. Apart from its use in defense, it is also applied in other fields, such as petroleum and natural gas industry.

The main mechanism of a shaped charge (SC) is acceleration and collapse of metal liner by the gaseous products of detonation. The collapse of the liner to the symmetry axis is followed by its plastic deformation and formation of high-velocity penetrator, a process that can be seen in Fig. 1. This jet consists from a secondary part – the slug, which is relatively slow and significantly faster primary jet, which is responsible for target penetration.

The representative parameters of a shaped charge jet are presented in Table 1.

The focus of this research is the jet penetration modeling. Although both analytical and numerical models offer insight into this complex process, each approach presents distinct challenges. Analytical models, while conceptually straightforward, often rely on simplifications that compromise accuracy. On the other hand, uncertainty or unavailability of material properties and high computational cost are the most important disadvantages of numerical models. Notably, the jet penetration phase imposes significantly greater computational demands compared to preceding processes of jet formation. This research aims at providing a deeper understanding of the jet interaction with target, as well as on determining its influence on penetration depth.

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Table 1. Characteristic values of a shaped charge warhead/projectile and jet [1,2]

SC cone diameter	d = (40 … 700) mm
Detonation velocity	D = (8.0 … 9.5) km/s
Jet diameter	≈ d/20, variable
Jet length	≤ 8d, variable
Hole diameter	≈ d/6
Jet tip velocity	≈ D (up to 10 km/s)
Jet tail velocity	(2…3) km/s
Penetration depth	(5…10) d

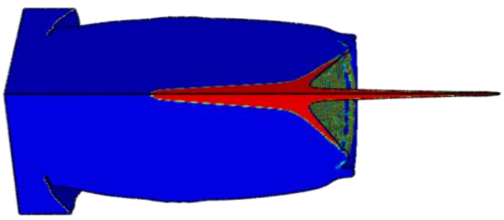

Figure 1. Numerical simulation of shaped charge effect: a sequence of the process of liner collapse and jet formation

Analytical models

Two most prominent analytical models will be briefly described: the well-known density law and the model with variable jet velocity.

Density law

The main assumption of the density law model is that the jet velocity v_j is uniform, i.e. jet length is constant (Fig. 2). Additionally, a dominant penetration mechanism is erosion of both the target plate and jet, due to extremely high contact pressure which exceeds the material yield stress by two orders of magnitude. As a consequence, contact surface between the target and the jet moves with velocity u which is known as penetration velocity. Applying the pressure equilibrium equation in the moving coordinate system, the relation between the two aforementioned velocities can be established:

$$u = \frac{v_\mathrm{j}}{1+\gamma}, \quad \gamma = \sqrt{\frac{\rho_\mathrm{t}}{\rho_\mathrm{j}}}, \tag{1}$$

where ρ_t and ρ_j are the target and jet material densities, respectively. The density law is based on assumption that the penetration process is completed when the entire jet is consumed by erosion. The penetration depth P may be then calculated using Eq. (2), from which is evident that its value is proportional to the jet length and the ratio between densities of the jet and the target material:

$$P = l_\mathrm{j}\sqrt{\frac{\rho_\mathrm{j}}{\rho_\mathrm{t}}} = \frac{l_\mathrm{j}}{\gamma}. \tag{2}$$

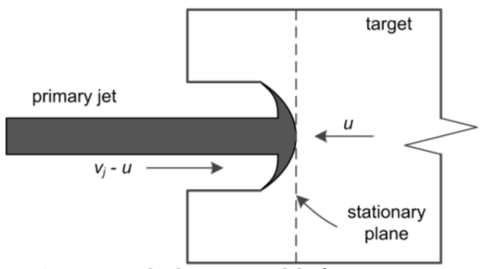

Figure 2. Hydrodynamic model of jet penetration

Penetration depth of variable velocity jet

The second, more complex model takes into account the fact that jet velocity is not uniform, due to which the jet gets elongated. This approach is based on the concept of the virtual origin [5], [6]. This concept abstracts the jet formation process, assuming that all jet particles are simultaneously ejected with different velocities from the virtual origin (Fig. 3). Similar treatment of equilibrium and erosion, and more complex approach to penetration dynamics lead to the final expressions for the current jet tip velocity

$$v_\mathrm{j} = v_0\left(\frac{s}{s+p}\right)^{\gamma} \tag{3}$$

and final penetration depth

$$P = s\left[\left(\frac{v_0}{v_{\min}}\right)^{\frac{1}{\gamma}} - 1\right]. \tag{4}$$

In previous equations v_0 is the jet tip velocity, p is the current penetration depth, s is the effective standoff – the distance from the virtual origin to the target and $v_{\min}$ designates the minimum jet velocity under which the penetration may be achieved through the mechanism of erosion. This value can be empirically calculated from:

$$v_{\min} = (1+\gamma)u_{\min}, \quad u_{\min} = 0.44 + 0.00206 \cdot BHN. \tag{5}$$

where BHN is the Brinell hardness number for target material.

This model takes also into account more complex cases when jet particulation occurs and the jet break-up time can be involved in corresponding variants of Eq. (4) [5], [6].

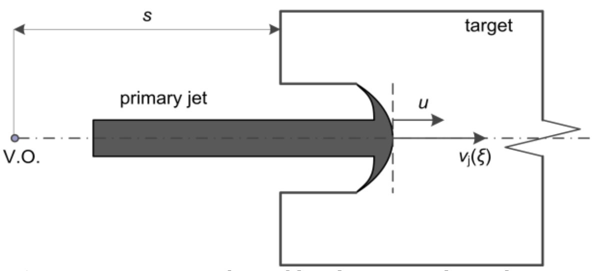

Figure 3. Penetration with variable velocity jet and virtual origin concept

Jet kinetic energy and crater volume

A well-know hypothesis in terminal ballistics about the link between the jet kinetic energy and the volume of the crater formed in the target [1], [4] will be investigated. Assuming a linear jet velocity profile in accordance with the virtual origin concept, the kinetic energy of the jet can be expressed as:

$$E_\mathrm{k} = \frac{1}{2}m_\mathrm{j}v_\mathrm{j}^2 = \frac{1}{6}m_\mathrm{j}v_0^2\left[1+\frac{v_t}{v_0}+\left(\frac{v_t}{v_0}\right)^2\right] \tag{6}$$

where m_j is the mass of the primary jet, and v_0, v_t and v_j are the jet tip, jet tail, and average jet velocities, respectively.

Assuming that the average representative value of the cylindrical crater radius is r_c, the crater volume is:

$$V_\mathrm{c} = \pi r_\mathrm{c}^2 P \tag{7}$$

The Szendrei-Held model [7-10] has been proposed to describe the radial expansion of the target material based on the equation of motion, alongside with previously considered axial crater growth. The dynamics of the crater radial expansion can be described by:

$$\frac{\mathrm{d}r_c}{\mathrm{d}t} = \sqrt{\frac{A}{r_c^2} - B}\,, \tag{8}$$

where parameters A and B are defined through:

$$A = \frac{1}{2}\frac{\rho_j}{\rho_t}\frac{\gamma^2}{(1+\gamma)^2} r_j^2 v_j^2\,, \quad B = \frac{\sigma}{\rho_t}\,. \tag{9}$$

In the previous equation r_j denotes the jet radius, while σ is the average dynamic plastic stress of the target material. Therefore, the maximum value of the crater diameter can be calculated:

$$r_c = r_{c,\max} = \sqrt{\frac{A}{B}} = r_j \frac{1}{1+\gamma}\sqrt{\frac{\rho_t v_j^2}{2\sigma}}\,. \tag{10}$$

Using Eqs. (6) – (10) one can calculate the ratio between input kinetic energy of the jet and the resulting created crater volume in the form:

$$\frac{E_k}{V_c} = \left(\frac{1+\gamma}{\gamma}\right)^2 \frac{l_j}{P}\sigma = \frac{(1+\gamma)^2}{\gamma}\sigma = \left(2+\gamma+\frac{1}{\gamma}\right)\sigma\,. \tag{11}$$

It should be noted that for typical practical applications (e.g. copper jet and steel target) density ratio γ is close to unity, so the right-hand side of Eq. (11) is approximately equal to 4σ. Obviously, the energy – volume ratio has the unit of pressure (or stress) and can be considered as a parameter of target resistance to jet penetration.

Referring to the well established Tate – Alekseevskii model [11] for jet penetration:

$$\frac{1}{2}\rho_j (v_j - u)^2 = \frac{1}{2}\rho_t u^2 + R\,, \tag{12}$$

target resistance R can be interpreted as the energy-volume ratio calculated by Eq. (11) [10]. Therefore, we have the following relation between the measures of axial (R) and radial (σ) crater expansion resistance:

$$R = 4\sigma\,. \tag{13}$$

Numerical approach

Numerical investigation of shaped charge jet penetration is performed using the FEM-based platform Abaqus [12] with its explicit solver, which has been successfully applied for various problems related with transient, high-energy and high strain rates phenomena, including shaped charge formation and penetration [13-15].

Having in mind large plastic deformation of both the jet and the target materials, the Eulerian framework is preferred. In the present research, the focus was set on the penetration phenomena, so the jet formation phase was not considered. Instead, an already formed simplified jet of known material, geometric and kinetic properties was analyzed (Fig. 4). The target was considered to be semi-infinite and sufficiently large values of radius (60 mm) and thickness (from 800 mm to 1100 mm) were selected. The quarter-symmetry was utilized and the mesh which consists from about 2.5 million finite elements was created. Although the mesh was not uniform, the referent element size was equal to 0.75 mm.

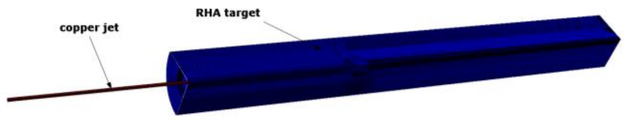

Figure 4. Model of the shaped charge copper jet impacting the RHA target

Definition of material behavior under the highly dynamic loads is of vital importance for the quality and precision of the numerical model. The considered shaped charge jet is made from copper and the target plate from rolled homogeneous armor steel (RHA). Dynamic plasticity of both metals is described by the Johnson-Cook plasticity model [16] and linear shock wave relation. Material failure is treated by the Johnson-Cook damage model [17]. The material properties used in Abaqus simulations have been summarized in Table 2.

Table 2. Material properties for jet material (copper) and target material (RHA) [16, 17]

	Copper	**RHA**
density (kg/m^3)	8960	7850
Johnson-Cook plasticity model		
A (MPa)	90	1400
B (MPa)	292	1800
C	0.025	0.0049
n	0.31	0.768
m	1.09	1.17
T_{melt} (K)	1356	1800
T_{trans} (K)	300	300
Linear U_s – u equation of state		
c_0 (m/s)	6940	4578
s	1.49	1.33
Γ	2.0	1.67
Johnson-Cook damage model		
d_1	0.54	0.10
d_2	4.89	3.44
d_3	3.03	2.12
d_4	0.014	0.002
d_5	1.12	0.61

The nominal characteristics of the baseline jet in the moment of impact into the target are shown in Table 3. As the idea of the research was to investigate the influence of the jet properties on the penetration depth P and the volume of created crater V_c, the jet's kinetic energy was held constant, while two effects have been considered: (i) variation of the jet velocity profile (gradient), and (ii) variation of the jet effective standoff.

Table 3. Main nominal properties of the benchmark jet-target system

Property (unit)	Value
Jet diameter (mm)	6
Jet length (mm)	250
Jet tip velocity (m/s)	9200
Jet tail velocity (m/s)	3000
Mass of the jet (g)	63.3
Kinetic energy of the jet (MJ)	1.279
Target thickness (mm)	800 ... 1100

Results and discussion

Typical evolution of the baseline jet's penetration into the RHA target is shown in Fig. 5. In this case, the penetration depth was equal to 536 mm, while the time of penetration (from jet impact to the completion of the process) had the value of 232 μs.

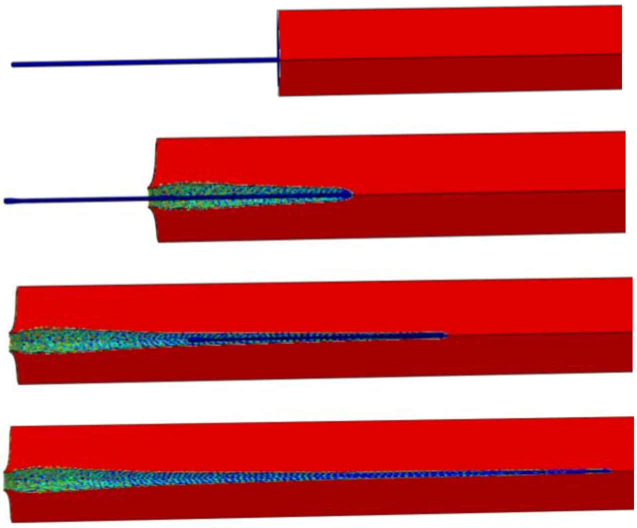

Figure 5. Evolution of the baseline jet's penetration into the RHA steel target. Jet material is in blue and target material in red color. Sequences correspond to the following moments: 0 μs, 40 μs, 120 μs and 232 μs. Penetration depth is 536 mm.

Penetration depth is investigated for varying velocity gradient of the jet – six variants of jet velocity profiles have been analyzed: from uniform velocity jet (v_0=v_t=6356 m/s) to the jet with intense velocity gradient (v_0=10056 m/s, v_t=2000 m/s). Kinetic energy of the jet was kept constant for each variant. Results are presented in Fig. 6 where the abscissa Δv corresponds to the difference between the jet tip and jet tail velocity. It can be seen that penetration depth increases with the increase in jet velocity gradient, reaching the maximum for $\Delta v \approx 5500$ m/s. Acceptable correspondence between numerical and analytical model results can be observed.

Relative penetration depth was also analyzed as a function of the effective standoff – the distance between the virtual origin and the target. Both numerical and analytical results indicate an increase of penetration depth with increasing standoff, as shown in Fig. 7. Models obviously fail to capture a well-known fact from experiments that there is an optimum standoff with maximum penetration capability of the jet. This is the consequence of the fact that both models assume ideal jets with perfect material homogeneity, geometric precision, symmetry and undisturbed break-up.

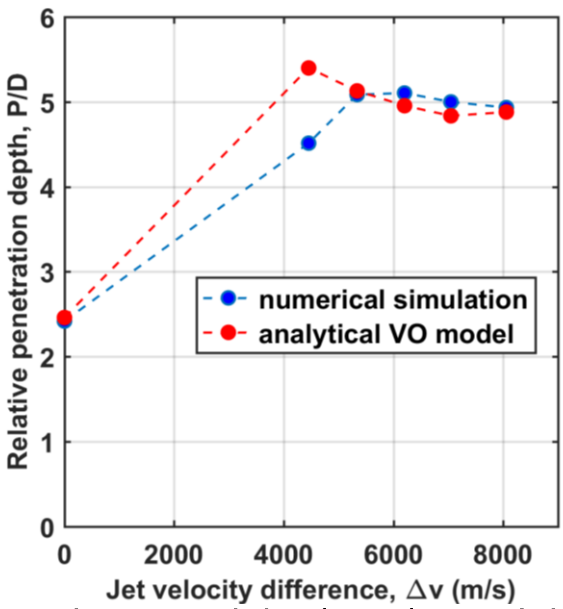

Figure 6. Relative penetration depth as a function of jet tip-to-tail velocity difference: comparison between results of numerical simulations and analytical model based on the virtual origin concept

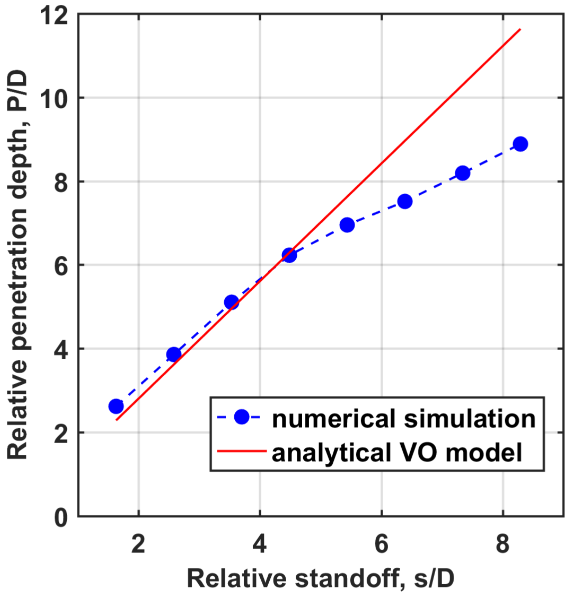

Figure 7. Relative penetration depth as a function of the effective standoff: comparison between simulations and analytical model

Volume of the crater created by the jet's penetration can be presented in the similar manner, as a function of both jet gradient Δv and relative standoff distance, which is shown in Figs. 8 and 9, respectively. Although a scatter in the data is obvious, it is clear that the values of crater volume are within ±10% deviation from the indicated avarage values. It should be noted that the exact values of crater volume cannot be obtained directly from the Abaqus output database due to the usage of Eulerian elements. Therefore, volumes are calculated through the use of a special image processing software ImageJ [18], which may be the source of additional discrepancies. Nevertheless, the hypothesis of approximatly constant crater volume produced by the jets of the same kinetic energy is considered to be confirmed.

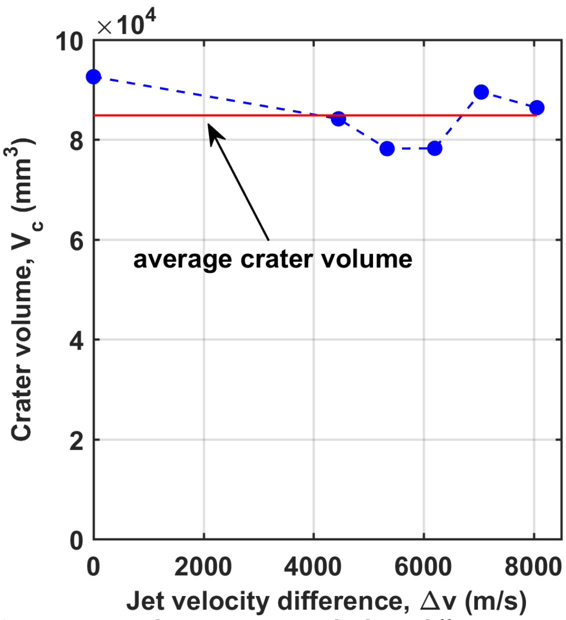

Figure 8. Crater volume vs. jet tip-to-tail velocity difference; average crater volume is indicated

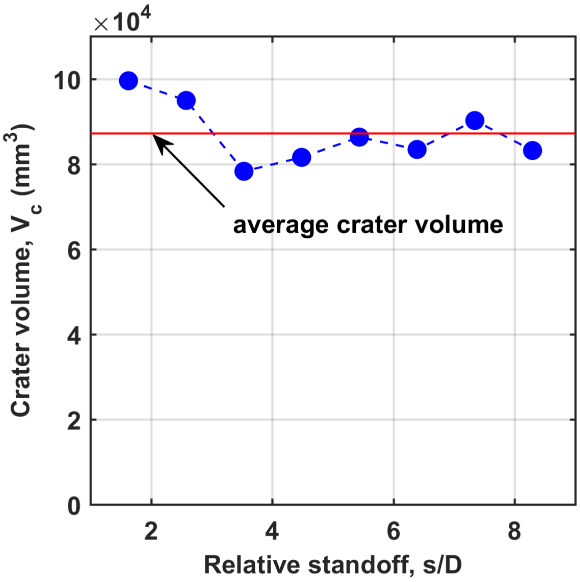

Figure 9. Crater volume vs. relative standoff distance; average crater volume is indicated

Moreover, the average crater volume from all simulations was equal to 86181 mm^3, and by applying the constant kinetic energy term from Table 3, one can determine the specific energy needed for cratering, i.e. characteristic material strength:

$$\frac{E_\mathrm{k}}{V_\mathrm{c}} = 14.833 \ \frac{\mathrm{J}}{\mathrm{mm}^3} = 14.833 \ \mathrm{GPa} \,. \tag{14}$$

On the other hand, the average representative value of the yield stress of the target material can be determined from the Johnson-Cook diagram (Fig. 10) using the data from Table 2. Having in mind that simulations indicate the maximum equivalent plastic strain of 2.8, and by neglecting the effects of strain rate hardening and thermal softening or considering them to be self-balancing, the average flow stress of σ=3.645 GPa can be calculated. According to Eq. (13), the target resistance can be found:

$$R = 4\sigma = 14.580 \ \mathrm{GPa} \,. \tag{15}$$

The values obtained by Eqs. (14) and (15) are remarkably close, confirming the link between the specific energy required for cratering and the target material dynamic flow stress.

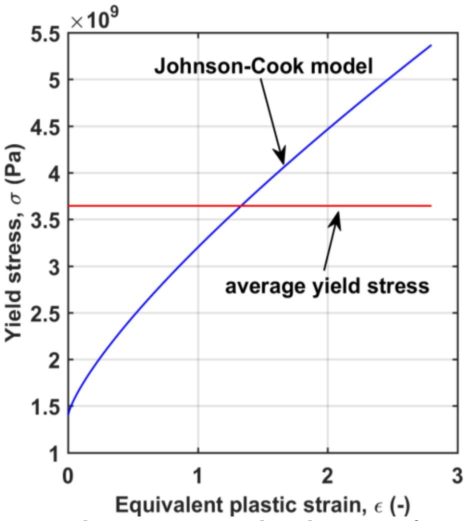

Figure 10. Flow stress vs. equivalent plastic strain for RHA steel according to the Johnson-Cook model; the average flow stress for the plastic strain of up to 2.8 is indicated

Conclusion

The paper considers various aspects of shaped charge jet penetration modeling. The following conclusions can be drawn:

– numerical simulations in Abaqus/Explicit provide detailed insight into the penetration process,

– jet characteristics, such as the velocity gradient and the standoff distance, significantly influence both the penetration depth and the volume of created crater,

– analytical penetration models exhibit limitations in terms of treatment of jet non-ideality (imperfect symmetry, break-up drift, etc.),

– crater volume created by the jet can be evaluated from the jet kinetic energy and the target material flow stress,

– further investigation can be focused on various issues, including jet break-up modeling and jet non-ideality.

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References

[1] WALTERS, W.P., ZUKAS, J.A.: *Fundamentals of shaped charges*, John Wiley & Sons, New York, 1989.

[2] ZUKAS, J.A., WALTERS, W.P.: *Explosive effects and applications*, Springer, New York, 2002.

[3] CARLUCCI, D.E., JACOBSON, S.S.: *Ballistics: Theory and design of guns and ammunition*, CRC Press, Boca Raton, FL, 2018.

[4] ELEK, P.: *Terminal ballistics* (in Serbian), University of Belgrade – Faculty of Mechanical Engineering, 2018.

[5] ALLISON, F.E., VITALI, R.: *A new method for computing penetration variables for shaped-charge jets*, Ballistic Research Laboratories, Aberdeen Proving Ground, Maryland, 1963.

[6] DIPERSIO, R., SIMON, J., MERENDINO, A.: *Penetration of Shaped-Charge Jets into Metallic Targets*, BRL Report No. 1296, 1965.

[7] SZENDREI, T.: Analytical model of crater formation by jet impact and its application to calculation of penetration curves and hole profiles, Proceedings of 7th International Symposium on Ballistics, The Hague, 1983, pp. 575-583.

[8] SZENDREI, T.: Analytical model for high-velocity impact cratering with material strengths: Extensions and validations, 15th International Symposium on Ballistics, Jerusalem, 1983, pp. 123-131.

[9] SZENDREI, T.: *Link between axial penetration and radial crater expansion in hypervelocity impact*, 17th International Symposium on Ballistics, Midrand, 1998, pp. 3-25-3-32.

[10] HELD, M.: Verification of the equation for radial crater growth by shaped charge jet penetration, *International Journal of Impact Engineering*, Vol. 17, 1995, 387-398.

[11] WALKER, J.: *Modern impact and penetration mechanics*, Cambridge University Press, Cambridge, 2021.

[12] DSSC, *Abaqus Theory Manual*, Dassault Systemes, Simulia Corp, Providence, RI, USA, 2023.

[13] DEHESTANI, P., FATHI, A., DANIALI, H.M.: Numerical study of the stand-off distance and liner thickness effect on the penetration depth efficiency of shaped charge process, *Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science*, 233(3) (2019) 977-986.

[14] ELEK, P., MARKOVIĆ, M., JEVTIĆ, D., ĐUROVIĆ, R.: *Modeling of penetration depth of a shaped charge jet*, 9th Congress of the Serbian Society of Mechanics, Vrnjačka Banja, Serbia, July 5-7, 2023.

[15] ELEK, P., MARKOVIC, M., JEVTIC, D., DJUROVIC, R.: *Numerical and analytical modeling of a shaped charge penetration depth*, Proceedings of the 4th International Conference on Impact Loading of Structures and Materials, Freiburg, Germany, 13-17 May 2024, pp. 88-89.

[16] JOHNSON, G.R., COOK, W.H.: A constitutive model and data for metals subjected to large strains, high strain rates and high temperatures, Proceedings of the 7th International Symposium on Ballistics, The Hague, 1983.

[17] JOHNSON, G.R., W.H. COOK: Fracture characteristics of three metals subjected to various strains, strain rates, temperatures and pressures, *Engineering Fracture Mechanics,* 21 (1985) 31–48.

[18] FERREIRA, T., RASBAND, W.: ImageJ User Guide IJ 1.46r, imagej.net (accessed: 10/07/2024)

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Modeliranje dubine prodiranja kumulativnog mlaza: Analitički i numerički pristup

Kumulativni efekat predstavlja najefikasniji mehanizam za probijanje oklopa, koji koristi energiju detonacije eksplozivnog punjenja za formiranje i ubrzavanje hiperbrzog metalnog penetratora poznatog kao kumulativni mlaz. Kumulativni efekat ima složenu dinamiku koja uključuje prostiranje detonacionog talasa, njegovu interakciju sa metalnom oblogom, a zatim urušavanje i preoblikovanje obloge što dovodi do formiranja kumulativnog mlaza. Ovaj složeni proces se može modelirati analitički i numerički, pri čemu oba pristupa imaju određena ograničenja. Analitički modeli, iako su konceptualno jednostavni, često se oslanjaju na pojednostavljenja koja utiču na tačnost. Sa druge strane, nepouzdanost ili čak nedostupnost relevantnih karakteristika materijala i značajno računarsko vreme su najvažniji nedostaci numeričkih modela. Značajno je da faza probijanja, tj. interakcije kumulativnog mlaza sa preprekom nameće znatno veće zahteve u pogledu računarskog vremena u poređenju sa procesom formiranja mlaza. Ovo istraživanje ima za cilj da pruži dublje razumevanje interakcije mlaza sa preprekom, kao i da razmotri uticaj karakteristika mlaza na dubinu prodiranja. Razmatran je analitički model zasnovan na konceptu virtuelnog ishodišta, dopunjen numeričkim simulacijama razvijenim u programskom paketu Abaqus/Explicit u čisto Euler-ovom domenu. Kroz sveobuhvatnu analizu, razmotreni su različiti parametri kumulativnog mlaza – kao što su kinetička energija, prečnik, dužina, gradijent brzine i efektivni standoff – i njihov uticaj na dubinu prodiranja. Uvidi dobijeni iz ove studije imaju praktičan značaj za preliminarnu procenu efikasnosti i unapređenje konstrukcije projektila, odnosno bojnih glava na bazi kumulativnog efekta.

Ključne reči: **kumulativni efekat, formiranje kumulativnog mlaza, probijanje kumulativnog mlaza, dubina prodiranja, numeričke simulacije.**