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Fatigue Life Analysis of Aircraft Structural Components

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This work defines an effective computation procedure that combines Neuber`s Rule and the finite element method with strain–life criterions in order to accurately predict fatigue crack initiation life and then establish an estimated schedule of fatigue life. The proposed procedure is applied to the representative aircraft structural components such as a plate with hole to obtain predicted lives. Computation results are compared with experimental results. Comparative results demonstrate that the fatigue life estimated by the novel procedure closely approximates the experimental results.

Key words: **fatigue, aircraft, aircraft structures, scyclic loading, crack initiation, finite element method, numerical simulation.**

Introduction

SEVERAL different analysis procedures are currently available for use in uniaxial fatigue life evaluations. The procedures can be broadly classified as either "crack initiation" or "crack propagation approaches".In the recent years it has been recognized that the fatigue failure process involves three phases. A crack initiation phase occurs first, followed by a crack propagation phase; finally, when the crack reaches a critical size, the final phase of unstable rapid crack growth to fracture components, the failure process. The modeling of each of these phases has been under intense scrutiny, but the models have not yet been developed in a coordinated way to provide a widely accepted engineering design tool. Fatigue design against crack initiation may lead to different material selection criteria and structural design from fatigue design against crack propagation.

The aim of the study is to define a complete procedure for fatigue life prediction of structural elements up to crack initiation. The procedure for fatigue life estimation is based on combining computation stress analysis with strain-life methods. Methods for stress analysis that will be used here are analytical and FE method. The analytical method proved to be easier while FEM is adequate for application with complex structures. FEM is favorable for detection of critical locations. Additionally, FEM is a reliable method for the stress analysis, both for linear and elastic-plastic domain. Material behavior and analytical description of cyclic curve is analyzed and included during fatigue life estimation for structural elements up to crack initiation. During fatigue life estimation of the structural elements it is necessary to adopt and use adequate criteria up to crack initiation, as it was done here.

Fatigue Life Prediction Based on Local Stress-Strain Concept

Although progress has been much slower in modeling the crack initiation phase, the most promising approach to the prediction of crack initiation seems to be the local stress-strain concept. The basic premise of the local stress-strain approach is that the local fatigue response of the material at the critical point, that is, the site of crack initiation, is analogous to the fatigue response of a small, smoth specimen subjected to the same cyclic strains and stresses[14]. The cyclic stress-strain response of the critical material may be determined from the characterizing smooth specimen through appropriate laboratory testing. To properly perform such laboratory tests, the local cyclic stress-strain history at the critical point in the structure must be determined, either by analytical or experimental means, thus, valid stress analysis procedures, finite element modeling, or experimental strain measurements are necessary, and the ability to properly account for elastic-plastic behavior must be included. In performing smooth specimen tests of this type, it must be recognized that the phenomena of cyclic hardening, cyclic softening, as well as sequential loading accumulates fatigue damage presumed to be the same as at the critical point in the structural component being simulated. Some data have been accumulated to support the validity of this postulate [14,15].

The strain-life method may be summarized as follows [4]:

1. By means of linear static finite element analysis (FEA), derive the local stress-strain time history from the load time history. This includes super positioning of multiple FEA/load time history load cases.
2. Extract the fatigue cycles in the local stress time history by means of the rain flow algorithm.
3. Make the elastic-plastic correction using Neuber's rule or nonlinear FEA.
4. Model the fatigue crack initiation process using hysteresis loop simulation based on the cyclic stress-strain curve.
5. Assess the damage contribution of each closed hysteresis loop by referring to the selected damage curve (strain-life, Morrow, SWT curves).

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6. Linearly sum the damage associated with each cycle by using Miner's rule.

One of the important aspects of the study of fatigue is a never-ending search for simple experimental or analytical methods [4] that can be used for the prediction of fatigue behavior in relatively complex situations. A good example of this approach was the use of monotonic (cyclic) test data in attempting to predict the fatigue life. Even for the simplest and most ideal specimen geometry and loading pattern such a prediction is a substantial step forward. In a similar manner we can conceive the prediction of fatigue behavior in a notched structural element on the basis of known cyclic behavior in a simpler smooth specimen. Recent advances in this area have produced interesting results and raise the hope of having fairly useful methods in the future [5,10].

Stress is determined through analytical estimation and with application of FEM. For the analytical estimation, Neuber's rule was used to predict the notch strain and stress amplitudes. After stress determination, fatigue life estimation was evaluated for structural elements with stress concentrations for constant amplitude loading and variable amplitude loading. During their working life, structural elements are subject to complex loading; i.e. multi-axial loading and they have to be analyzed. Single axial fatigue has greatly developed for long time such as local stress method and energy method. Now most of the method is to equalize the multi-axial fatigue damage to single axial damage style, and then use the single axial fatigue damage theory to predict multi-axial fatigue life. Multi-axial fatigue damage model can be divided into three classes, which is based on the following criterion: one is the largest main stress (strain), the other is von Misses criterion and the last is Tresca criterion. In this paper the von Misses criterion will be explained.

Von Misses Criterion

With von Mises criterion, the axial strain and the shear strain can be composed to equivalent strain amplitude:

$$\varepsilon_{eq,a} = \frac{1}{(1+\nu)\sqrt{2}}\Big(\left(\varepsilon_{1,a}-\varepsilon_{2,a}\right)^2 + \left(\varepsilon_{2,a}-\varepsilon_{3,a}\right)^2 + \\ +\left(\varepsilon_{3,a}-\varepsilon_{1,a}\right)^2\Big)^{1/2} \quad (1)$$

In formula (1) $\varepsilon_{1,a}$, $\varepsilon_{2,a}$, $\varepsilon_{3,a}$ mean three main strains of simple axial loading, ν is the Poisson's ratio.

The next important thing to find is the equivalent strain range $\Delta\varepsilon_{eq}$ From the ASME Boiler and Pressure Vessel code Procedure [10] which is based on the von Misses hypothesis, it is possible to get equivalent strain range $\Delta\varepsilon_{eq}$.

$$\Delta\varepsilon_{eq} = \frac{1}{(1+\nu)\sqrt{2}}\Big(\left(\Delta\varepsilon_x-\Delta\varepsilon_y\right)^2 + \left(\Delta\varepsilon_y-\Delta\varepsilon_z\right)^2 + \\ +\left(\Delta\varepsilon_z-\Delta\varepsilon_x\right)^2 + 6\left(\Delta\varepsilon_{xy}^2+\Delta\varepsilon_{yz}^2+\Delta\varepsilon_{xz}^2\right)\Big) \quad (2)$$

Using the static yield theory of multi-axial loading and von Misses criterion, equivalent multi-axial stress range is:

$$\Delta S_{eq} = \frac{1}{2}\Big(\left(\Delta S_1-\Delta S_2\right)^2 + \left(\Delta S_2-\Delta S_3\right)^2 + \\ +\left(\Delta S_1-\Delta S_3\right)^2\Big)^{1/2} \quad (3)$$

In the formula (3) ΔS_1, ΔS_2, ΔS_3 mean three main stresses of single axial.

Cycle Stress – Strain Relation and Plastic Revision Under Multi-axial Loading

For an elastic – plastic material, subjected to cyclic loading, the stress and strain history will initially go through a transient state which asymptotes to a cyclic state. In this cyclic state, the behavior of the body can be divided in three alternative regions: 1. Elastic, 2. Elastic – plastic and 3. Failure.

The cyclic stress – strain curve is possible to obtain by connecting the tip of stable hysteresis loops for different strain amplitudes of fully reversed strain – controlled tests.

For cyclic multi-axial loading, the stable cycle stress – strain curve can be represented in analytical form by the Ramber – Osgood equation. The cycle stress – strain formula can be expressed as follows:

$$\Delta\varepsilon_{eq} = \Delta\varepsilon_{eq}^{e} + \Delta\varepsilon_{eq}^{p} = \frac{\Delta\sigma_{eq}}{E} + 2\left(\frac{\Delta\sigma_{eq}}{2K'}\right)^{\frac{1}{n'}} \quad (4)$$

In the formula (4), $\Delta\sigma_{eq}$, $\Delta\varepsilon_{eq}$ are the equivalent range of the local stress and strain of the multi-axial loading; E is Young's modulus; n' is the cyclic hardening exponent; K' is the cyclic strength coefficient; $\Delta\varepsilon_{eq}^{e}$ and $\Delta\varepsilon_{eq}^{p}$ mean equivalent elastic and plastic strain range, respectively.

The Stress Analysis

In general, stress analysis has a dual role to play in fatigue life assessments. The first part is to determine the critical locations, the areas most at risk in the components, so that subsequent service load measuring can be made most effectively. The extent of the stress analysis required can range from simple examination of a failed component through techniques such as brittle lacquer methods to finite element stress analysis. The second role of stress analysis is to overcome the very localized nature of fatigue. In engineering components it is often impossible to locate a strain gage at the critical site. The latter is generally at a stress concentration such as a fillet radius of a hole where the strain gradient can be very steep. The importance of stress concentrations was understood at a very early stage in the development of fatigue life prediction techniques. An accurate determination of stress is very important since it has a significant effect on fatigue life prediction up to the crack initiation. In this paper, structural elements with geometric discontinuities were considered and it is necessary to apply adequate methods for stress determination, which include stress concentrations. Stress determination will be defined by analytical method (Neuber`s rule) and with application of FEM. Both methods for stress determination were used since it is easier to use analytical method while in some cases it is necessary to use FEM. The analytical method is used for structural elements with simple geometry. The necessity to use FEM, in some cases, originates from the great complexity of geometry in the structural elements. The final development of stress analysis methods to be considered is the consideration of plasticity. Although most components used in the aircraft structures remain nominally elastic, at the critical location where failure will take place a plastic analysis has to be utilized. The finite element method can be effectively used to describe nonlinear stress-strain relationships.

The Analytical Method based on Neuber`s Rule

In loading of notched structural components, the highly stressed material is localized around the notch root. To compute local stresses and strains from external loading and geometry, Neuber`s rule is often used in conjuction with the cyclic stress-strain properties and properties and fatigue stress concentration factor. The Neuber`s rule may be written as [8]:

$$k_t = (k_\varepsilon k_\sigma)^{1/2} \tag{5}$$

where: k_t is theoretical stress concentration factor, $k_\sigma (= \sigma/S)$ is the local stress concentration factor $k_\varepsilon (= \varepsilon / e)$ is the local strain concentration factor, ε -local strain, e - nominal strain, σ -local stress, S -nominal stress. This relationship was modified for application to fatigue loading [15] by utilizing the fatigue stress concentration factor together with nominal stress range, ΔS, nominal strain range Δe, local stress range $\Delta\sigma$, and local strain range $\Delta\varepsilon$ to transform (5) into:

$$k_f (\Delta S\, \Delta e)^{1/2} = (\Delta\sigma\, \Delta\varepsilon)^{1/2} \tag{6}$$

If loads are small enough that the material behavior of the overall? member is nominally elastic, then $\Delta S / \Delta e = E$ and (6) becomes:

$$\frac{k_t^2\ \Delta S^2}{E} = \Delta\sigma\, \Delta\varepsilon \tag{7}$$

Since for a given loading and geometry, the left-hand side of eq (7) is known, this alternative form of Neuber`s Rule provides one relation between the two unknowns $\Delta\sigma$ and $\Delta\varepsilon$. Also, $\Delta\varepsilon$ is related to the $\Delta\sigma$ through the material stress-strain behavior. The two relations, Neuber`s Rule and material stress-strain law, completlely determine $\Delta\sigma$ and $\Delta\varepsilon$. Thus, using the cyclic stress-strain relation widely accepted for fatigue problems:

$$\Delta\sigma = \frac{\Delta\sigma}{E} + 2\left(\frac{\Delta\sigma}{2K'}\right)^{1/n'} \tag{8}$$

and substituting into Eq (7), one obtains an expression relating local stress range $\Delta\sigma$ to the applied nominal stress range ΔS and k_t from which $\Delta\sigma$ may be solved for

$$\begin{aligned} (\Delta\sigma)^{1/2}\left[\frac{\Delta\sigma}{E} + 2\left(\frac{\Delta\sigma}{2K'}\right)^{1/n'}\right]^{1/2} &= \\ = k_t (\Delta S)^{1/2}\left[\frac{\Delta S}{E} + 2\left(\frac{\Delta S}{2K'}\right)^{1/n'}\right]^{1/2} \end{aligned} \tag{9}$$

where E, K' and n' are material properties. The value of $\Delta\sigma$ so obtained is then substituted back into eq (8) to solve for $\Delta\varepsilon$. In the case of multi-axial loading, eq. (5) needs to be modified in such a way that all stresses and strains need to be replaced with equivalent stresses and strains, i.e.:

$$\Delta\sigma_{eq}\, \Delta\varepsilon_{eq} = \frac{k_t^2\ \Delta S_{eq}^2}{E} \tag{10}$$

where $\Delta\sigma_{eq}$ and $\Delta\varepsilon_{eq}$ are equivalent strain range notch root stress and strain range, and ΔS_{eq} and Δe_{eq} are equivalent strain range nominal stress and strain range, respectively.

The Stress Analysis based on Finite Element Method

It is well known that stresses analysis with application of FEM provides the most accurate results. For the stress analysis MSC/NASTRAN software [11] was used. It is necessary to use FEM in the cases where geometry of structural elements is complex as well as in those cases where loading is complex (multi-axial). For the stress determination non-linear (elasto-plastic) analysis was used, which provides the most accurate results. Additionally, it should be noted that in this non-linear (elastic-plastic) analysis, the cyclic curve for material behavior was used.

Obtained values of local stresses with application of FEM will be used for fatigue life estimation and with application of adequate criteria.

The Fatigue Life Analysis

For life estimation strain-based approach is used. In this approach, local strains at notches, σ and ε, are estimated and used as the basis of life predictions. The analytical method is based on low – cycle fatigue data in terms of the strain–life curve, as they are conveniently used to present the strain cycling resistance of materials by describing the endurance as a function of both an elastic and plastic strain amplitude. The relationship between the applied strain range and fatigue life under multi-axial loading is given by Morrow [7] equation:

$$\frac{\Delta\varepsilon_{eq}}{2} = \frac{\sigma_f' - \sigma_m}{E}\left(2N_f\right)^b + \varepsilon_f'\left(2N_f\right)^c \tag{11}$$

In formula (11), $\Delta\varepsilon_{eq}$ is equivalent strain range; b is Basquin′s coefficient; c is fatigue ductility exponent; σ_f' is Basquin′s fatigue strength coefficient; ε_f' is fatigue ductility coefficient and σ_m is local mean stress.

Another relationship proposed by Smith, Watson and Topper [9] is formulated as:

$$\frac{\sigma_{\max}\ \Delta\varepsilon_{\max}}{2} = \frac{\left(\sigma_f'\right)^2}{E}\left(2N_f\right)^{2b} + \varepsilon_f'\ \sigma_f'\left(2N_f\right)^{b+c} \tag{12}$$

Where $\sigma_{\max}$ is the local maximum stress on the $\Delta\varepsilon_{\max}$ plane, $\Delta\varepsilon_{\max}/2$ is the maximum local strain amplitude. The relation (12) is used for materials domaged by tensile loading. Multi-axial loading can be consisted of shear (and sometimes shear is dominated), in that case it is possible to use relation proposed by Fatemi and Socie [2]:

$$\frac{\Delta\gamma_{\max}}{2} = \frac{\tau_f'}{G}\left(2N_f\right)^{b_0} + \gamma_f'\left(2N_f\right)^{c_0} \tag{13}$$

Where $\Delta\gamma$, τ_f', γ_f', b_0, c_0 are the shear strain range, the shear fatigue strength coefficient, the shear fatigue ductility coefficient, the shear fatigue strength exponent, the shear fatigue ductility exponent, respectively. These properties can be obtained from torsion fatigue tests, or they can be estimated from uni-axial strain – life properties as: $\tau'_f \approx \sigma'_f \cdot \sqrt{3}$, $\gamma'_f \approx \varepsilon'_f \cdot \sqrt{3}$, $b_0 = b$ and $c_0 = c$.

Fatigue Cumulative Damage

Fatigue life evaluation of mechanical components under complex loading conditions is of great importance to optimize structural design, and improve inspection and maintenance procedures.

Under variable amplitude loading, every same stress – strain cycle make the same damage, and is independent of the place in the load spectrum. Fatigue damages under variable amplitude were estimated by Palmgren-Miner rule. The Miner $[6]$ law is adopted, the damage D is expressed as follows:

$$D = \sum_i \frac{n_i}{N_{fi}} \tag{14}$$

In the formula (14), N_{fi} is the cycle count at the time of failure under of axial loading, the value n_i is the actual cycle count at the adequate stress level. Then the block load spectrum T when the structure is failure can be expressed as follows:

$$T = \frac{1}{\sum_i \frac{n_i}{N_{fi}}}. \tag{15}$$

The above equation represents statements of the linear damage rules used by the local strain fatigue life predictions.

Numerical Examples

To illustrate previous computation fatigue life procedure various examples are included in this paper. These examples consider structural elements with notches under constant amplitude load and load spectrum. Crack initiation lives of notched specimens subjected to axial loading with the life prediction procedure described in the above section.The predicted and available experimental results of the crack initiation lives of notched specimens are compared.

Example 1: Initial Fatigue Failure Analysis of Plate with Hole Under Cyclic Loads

In this example, crack initiation fatigue life estimation was carried out. The structural element was subject to constant amplitude loading and variable amplitude loading. As a structural element we used the plane with central hole of SAE 1045 steel, axially loaded. Characteristics of used material are as follows:

b = - 0.067; K' = 812.53 MPa;
c = - 0.500; S_y = 390 MPa;
σ'_f = 805.93 MPa; S_u = 650 MPa;
ε'_f = 0.941; E = 2.1 10^5 MPa;

Type of material: SAE 1045.

Geometry characteristics of plate with central hole are: $w = 40$ mm ; $2R = 8$ mm ; $t = 6$ mm ; $L = 100$ mm . Based on known material characteristics it is possible to define the cyclic curve with application of eq.(4). Materials behavior can be presented by cyclic curve (Fig.1).

Table 1. Cyclic Stress – strain curve

No.	σ_{a} a [MPa]	ε_{a} a
1.	0	0
2.	50	$2.38{\cdot}10^{-4}$
3.	100	$4.761{\cdot}0^{-4}$
4.	150	$7.16{\cdot}10^{-4}$
5.	200	$9.66{\cdot}10^{-4}$
6.	250	$1.26{\cdot}10^{-3}$
7.	300	$1.71{\cdot}10^{-3}$
8.	300	$1.71{\cdot}10^{-3}$
9.	350	$2.56{\cdot}10^{-3}$
10.	400	$4.32{\cdot}10^{-3}$
11.	450	$7.96{\cdot}10^{-3}$
12.	500	$1.52{\cdot}10^{-2}$
13.	550	$2.861{\cdot}0^{-2}$
14.	650	$9.36{\cdot}10^{-2}$

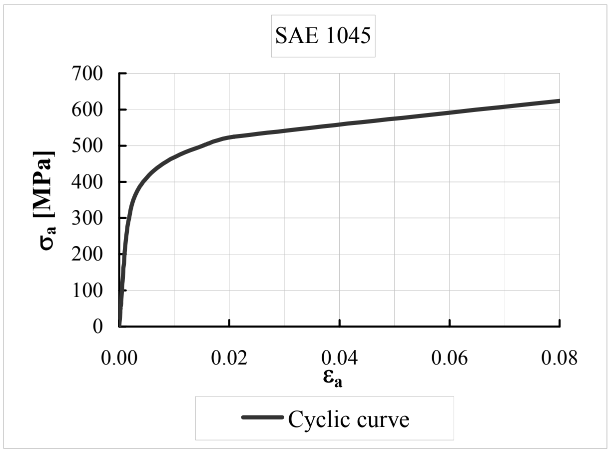

Figure 1. Cyclic curve σ_a - ε_a for steel SAE 1045

Using material cyclic stress – strain curve (4) and Neuber equation (eq.5), we can get the real stress and strain spectrum. Then using the fatigue performance data, according to the structure specialty and fatigue accumulated damage model, we can obtain the fatigue life of the structure. Here, we use Morrow criterion up to crack initiation. Obtained results were compared with available experimental results. The results are shown in Table 2 and Fig.2.

Table 2. Number of cycles up to crack initiation of the plate with central hole (SAE 1045, k_t = 2.5 i R = -1).

No.	P_{min} [kN]	P_{max} [kN]	S_{min} [MPa]	S_{max} [MPa]	S_a [MPa]	N_f [cycles]
1.	-33.6	33.6	-140	140	140	0.14688 10^7
2.	-36	36	-150	150	150	0.87262 10^6
3.	-48	48	-200	200	200	0.14077 10^6
4.	-54	54	-225	225	225	0.74616 10^5
5.	-60	60	-250	250	250	0.43854 10^5
6.	-66	66	-275	275	275	0.27742 10^5
7.	-72	72	-300	300	300	0.18541 10^5
8.	-78	78	-325	325	325	0.12933 10^5
9.	-84	84	-350	350	350	0.93354 10^4
10.	-96	96	-400	400	400	0.52728 10^4
11.	-108	108	-450	450	450	0.32342 10^4

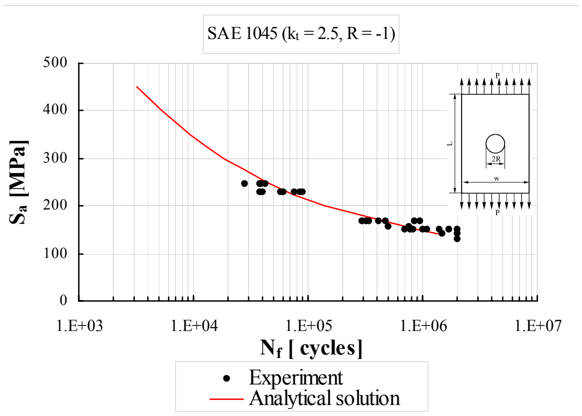

Figure 2. Fatigue life up to crack initiation of the plate with a central hole using analytical method (SAE 1045, $k_t = 2.5$ i $R = -1$, experiment[1])

Fig.2 shows the comparison of the experimental fatigue life against the predicted results. It is shown that the predictions with analytical approach have good accuracy by comparison with experimental results [1].The obtained number of cycles N_f is for constant amplitude loading. In this example we considered both constant and variable amplitude loading. Thus, for selected load spectrum, we carried out a number of blocks N_{b1} up to crack initiation estimation. The type of general load spectrum was presented in Fig.3.

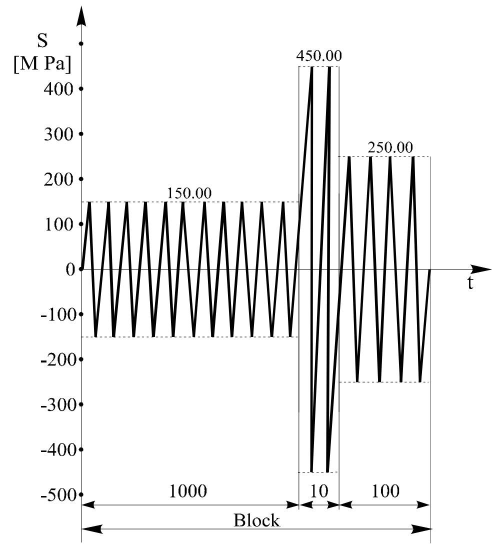

Figure 3. Load spectrum

First, the number of cycles up to crack initiation N_{f1} based on known characteristics of material and geometry was determined through calculation. Later, the number of blocks up to initial crack initiation N_{b1} using Miner's rule (eq.10).

Table 3. Number of cycles and blocks up to crack initiation for plate with the central hole (SAE 1045, $kt = 2.5$ i $R = -1$).

No	ni	S_{min} [MPa]	S_{max} [MPa]	S_a [MPa]	N_{fi} [cycles]	N_{b1}
1.	1000	-150	150	150	$0.87262\ 10^6$	
2.	10	-450	450	450	$0.32342\ 10^4$	
3.	100	-250	250	250	$0.43854\ 10^5$	$0.15432\ 10^3$

For load spectrum presented in Fig.3 the calculated values for the number of cycles up and requested number of blocks to crack initiation N_{f1} (eq.7) are listed in Table 3.

Example 2.: Initial Fatigue Life Predictions Based on FE stress Analysis

Similarly to ex. 1, fatigue life prediction up to initial failure of the plate with central hole was carried out. Both analytical method and FEM were used. External loading is axial with constant amplitude as well as variable amplitude. Obtained results were compared with available experimental results[12]. Material characteristics under cyclic loading are:

$b = -0.081$; $n' = 0.123$;
$c = -0.67$; $K' = 1062.1$ MPa;
$\sigma'_f = 1165.6$ MPa; $S_y = 648.3$ MPa;
$\varepsilon'_f = 1.142$ $S_u = 786.2$ MPa;
$E = 2.069\ 10^5$ MPa;

Geometry characteristics are: $w = 25.6$ mm; 2R = 12.8 mm; t = 7.68 mm; L = 100 mm. As in the previous example, the cyclic curve for material was obtained with application of eq.(4). Requested material curve (medium strength steel) is shown in Fig.4.

Table 4. Cyclic stress – strain curve

No.	σ_a [MPa]	$\varepsilon_{a\ a}$
1.	0	0
2.	50	$2.42\ 10^{-4}$
3.	100	$4.83\ 10^{-4}$
4.	150	$7.25\ 10^{-4}$
5.	200	$9.68\ 10^{-4}$
6.	250	$1.22\ 10^{-3}$
7.	300	$1.48\ 10^{-3}$
8.	350	$1.81\ 10^{-3}$
9.	400	$2.29\ 10^{-3}$
10.	450	$3.10\ 10^{-3}$
11.	500	$4.61\ 10^{-3}$
12.	550	$7.41\ 10^{-3}$
13.	600	$1.25\ 10^{-2}$
14.	650	$2.16\ 10^{-2}$
15.	700	$3.71\ 10^{-2}$
16.	750	$6.28\ 10^{-2}$
17.	800	$1.04\ 10^{-1}$
18.	850	$1.68\ 10^{-1}$

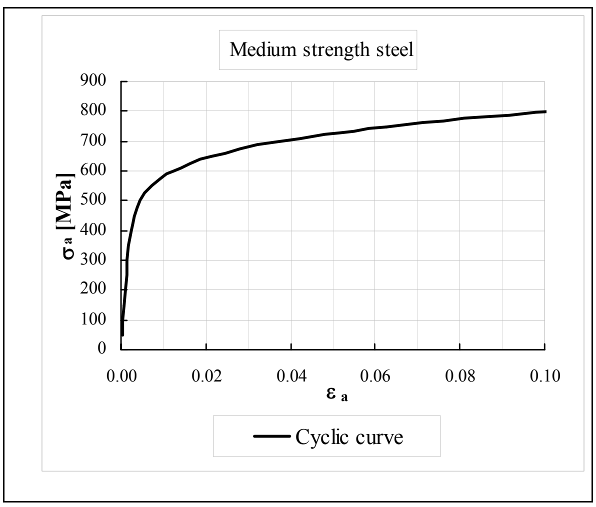

Figure 4. Cyclic curve σ_a - ε_a for medium strength steel

Now that we defined cyclic curve for material (eq.4) it is possible to start with fatigue life estimation. Since we are dealing with the plate with a central hole, it is necessary to determine local stresses first, so in this example FEM will be used. Results obtained with application of FEM, i.e. values of local stresses are presented in Table 6. In this paper we will present obtained stress distribution for a loading case 4 (Table 6). Fig.5 presents stress distribution for the plate with a central hole for selected loading level that appears within considered spectrum (Fig.7). In this case, axial force is P = 47390 N.

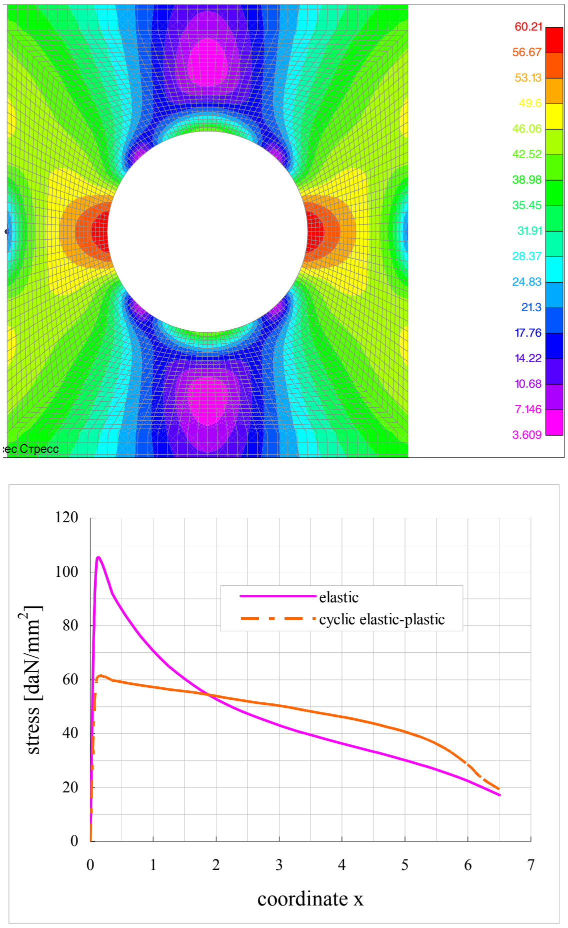

Figure 5. Stress distribution for the plate with a central hole

Based on the known characteristics of the material and geometry, calculated values of the local maximal stress (FEM) and the number of cycles up to crack initiation (using Morrow criterion) are presented in Table 5.

Table 5. Number of cycles up to crack initiation of the plate with a central hole (Medium strength steel, $R = -1$)

				N_f	[cycles]
No.	P_{max} [kN]	S_{max} [MPa]	σ_{max} (FEM) [MPa]	Experiment[3]	Predicted life (FEM)
1.	62.25	321.61	722.50	0.68000 102	$0.65840\ 10^2$
2.	56.29	290.85	671.90	$0.19000\ 10^3$	$0.15985\ 10^3$
3.	53.89	278.43	653.60	$0.26500\ 10^3$	$0.22475\ 10^3$
4.	47.39	244.84	602.10	$0.12500 10^4$	$0.61964\ 10^3$
5.	40.18	207.60	550.30	$0.24000\ 10^4$	$0.19216\ 10^4$
6.	40.14	207.39	550.10	$0.36000\ 10^4$	$0.19213\ 10^4$
7.	31.14	160.91	485.90	$0.11500\ 10^5$	$0.95469\ 10^4$
8.	25.27	130.56	439.40	$0.55400\ 10^4$	$0.37459\ 10^5$
9.	22.02	113.80	407.00	$0.16078\ 10^6$	$0.11106\ 10^6$
10.	20.92	108.08	394.60	$0.18800\ 10^6$	$0.17367\ 10^6$

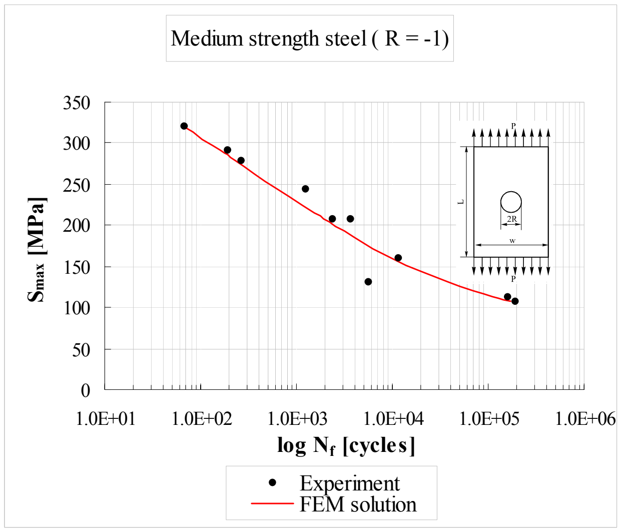

Figure 6. Fatigue life up to crack initiation of the plate with a central hole using FEM (Medium strength steel, $R = -1$, experiment[3]).

Fig.6 shows the comparison of the experimental fatigue life against the predicted results using FEM. The results show that FEM gives conservative solution.

Again, in this example, we will evaluate the number of blocks up to crack initiation for the load spectrum shown in Fig.7. Thus, we will consider variable amplitude loading.

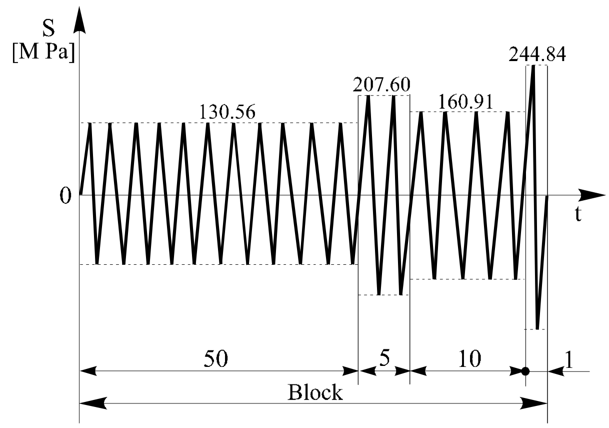

Figure 7. Load spectrum

For load spectrum shown in Fig.7, the local stresses were determined for each level first (analytical approach (eq.7) and FEM). Afterwards the number of cycles up to crack initiation (eq.11) was determined.

Table 6. Number of cycles up to crack initiation for plate with the central hole (Medium strength steel, $R = -1$).

				Nfi	[cycles]	Nbl	
No.	ni	Pmax [kN]	Smax [MPa]	Analytical solution	FEM	Analytical solution	FEM
1.	50	25.27	130.56	$0.21399\ 10^5$	$0.37459\ 10^5$		
2.	5	40.18	207.60	$0.22140\ 10^4$	$0.19216\ 10^4$		
3.	10	31.14	160.91	$0.67311\ 10^4$	$0.95469\ 10^4$		
4.	1	47.39	244.84	$0.11901\ 10^4$	$0.61964\ 10^3$	$0.14449\ 10^3$	$0.151278\ 10^3$

Finally number of blocks Nbl up to crack initiation was calculated. All results are shown in Table 6.

Conclusion

This work defines an effective complete procedure to predict fatigue life up to crack initiation. The presented procedure includes into analysis cyclic elastic-plastic material behavior, nonlinear finite elements analysis and strain–life criteria. Additionally, this procedure considers the interaction between loads – time history. Miner's rule was used to calculate the accumulative damage in the fatigue crack initiation phase. This procedure is then applied to a plate with a central hole (structural elements with concentrations), and the results were compared with analytical local strain method and available experimental data. Comparative results demonstrate that the fatigue life estimated by the presented procedure closely approximates experimental results. Critical stress estimated with Neuber`s Rule in this work is in close agreement with results obtained by the finite element method. Compared with FEM, Neuber`s Rule is much easier to use for estimating stress concentration.

It is very important to stress that presented procedure for prediction of fatigue life up to crack initiation provides good correlation with experimental data even with low and high fatigue domains.

Additionally, the above-mentioned procedure can be used for complex structural elements (which contain geometric discontinuities). The defined procedure, for fatigue life prediction of notched aircraft structural components up to crack initiation, can take into consideration uni-axial and multi-axial loading with constant and variable amplitude.

References

[1] HAIBACH,E. and MATSCHKE,C.: *The Concept of Uniform Scatter Bands for Analyzing S–N Curves of Un-notched and Notched Specimens in Structural Steel, Low-Cycle Fatigue and Life Prediction*, ASTM STP 770, C. Amzallag, B. N. Leis and P. Rabbe, Eds., American Society for Testing and Materials, 1982, pp. 549-571.

[2] FATEMI,A. and SOCIE: *Fatigue and Fracture of Engineering Materials and Structures*, 11(3) 1988, pp.149–165.

[3] MAKSIMOVIĆ,S., BOLJANOVIĆ,S. and MAKSIMOVIĆ,K.: *Improved Numerical Procedure in Fatigue Life Prediction of Structural Components Under Variable Amplitude Loads*, IFC-8-Fatigue 2002, Stocholm, 2–7.June 2002, Vol.1, pp.675–682.

[4] MAKSIMOVIĆ,S.: *Efficient Method in fatigue Failure Analysis of Complex Structures Under Variable Amplitude Loading*, Int. Journal Communications in Dependability and Quality Management, 2001, Vol.4, No.2.

[5] MAKSIMOVIĆ,S.: MLADENOVIĆ,J., *"FAILURE-3" – Fatigue Analysis of Structures Subjected to General Load Spectra (in-house software)*, Internal Report VTI, Belgrade 1996.

[6] MINER,M.A.: *Cumulative Damage in Fatigue,* J. Appl. Mech., 12, 159-164, 1945.

[7] MORROW,J.: *Fatigue Design Handbook, Advances in Engineering*, Society of Automotive Engineers, Warendale, Pa., 1968, Sec.3.2, pp.21–29.

[8] NEUBER,H.: *Theory of Notch Stresses: Principles for Exact Stress Calculation*, trans. from the German F. A. Raven for the David Taylor Model Basin, U.S. Navy, 1945. Also published by Edwards Brothers, Ann Arbot, Michigan, 1946.

[9] SMITH,R.N., WATSON,P. and TOPPER,T. H.: *A Stress – Strain Function for the Fatigue Materials*, J. of Materials, 5(4), pp. 767-778, 1970.Wetzel, R. M., *Smooth Specimen Simulation of the Fatigue Behavior of Notches*, Journal of Materials, Sept. 1968, Vol.3, No.3, pp.646–657.

[10] ASME Code Case N-47-23 (1988) Case of SME Boiler and Pressure Vessel Code, American Society of Mechanical Engineers.

[11] MSC/NASTRAN Manual

[12] MAKSIMOVIĆ,S. and JANKOVIĆ,M., *Life Estimation of Structural Components at the Low-Cycle Fatigue*, IRMES`04, Kragujevac.

[13] BOLJANOVIĆ,S., MAKSIMOVIC,S., *Initial Fatigue Life Predictions of a Notched Structural Components Under Variable Amplitude Loading*, The First Int. Conference on COMPUTATION MECHANICS, Belgrade, 15-17 November 2004, Eds. D. Mijuca and S. Maksimović.

[14] MORROW,J.D., MARTIN,J.F., and DOWLING,N.E.: *Local Stress-Strain Approach to Cumulative Fatigue Damage Analysis,* Final Report, T.& A. M. Report No. 379, Department of Theoretical and Applied Mechanics, University of Illinois, Urbana, January 1974.

[15] TOPER,T.H., WETZEL,R.M., and MORROW,J.: *Neuber`s rule Applied to Fatigue of Notched Specimens*, Journal of Materials, 1969, Vol.4, No.1.

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Procena veka elemenata avionskih konstrukcija

U okviru rada definiše se pogodna proračunska procedura, koja se bazira na kombinaciji Nojberovog zakona i metode konačnih elemenata sa kriterijumima na bazi deformacija-vek, kako bi se obezbedilo tačno predviđanje zamornog veka do pojave inicijalnog oštećenja a potom uspostavio kompletan pristup za procenu zamornog veka. Predložena procedura je primenjena na reprezentativni strukturalni element kao što je ploča sa otvorom opterećena cikličnim opterećenjima da bi se odredio njen proračunski vek. Proračunski rezultati su upoređeni sa eksperimentima. Procena veka primenom predložene proračunske procedure je u dobroj saglasnosti sa eksperimentalnim rezultatima.

Ključne reči: **zamor materijala, avion, struktura letelice, ciklično opterećenje, inicijalna prskotina, metod konačnih elemenata, numerička simulacija**

Estimation de la durée de vie des constructions d'avions

Ce travail définit un procédé de calcul efficace basé sur la combinaison de la loi de Nojber et de la méthode des élements finis. Les critères en sont basés sur la déformation de la durée de vie dans le but d'assurer une prévision précise de la durée de fatigue jusqu'à l'apparition de la défaillance initialle et pour établir une approche complète concernant l'estimation de la fatigue du matériel.Le procédé proposé est appliqué sur un élement structural représentatif. Il s'agit d'une plaque avec ouverture chargée de charge cyclique pour déterminer la durée de son calcul. Les résultats de ce calcul ont été comparés avec les essais. L'estimation de la durée de vie par application du procédé de calcul est en bon accord avec les résultats d'essais.

Mots clés: **fatigue de matériel, avion, structure d'aéronef, charge cyclique, fissure initialle, méthode d'élements finis, simulation numérique**

Оценка срока службы (ресурса) составных частей конструкции самолетов

В рамках этой работы определяется подходящая расчетная методика, которая базируется на комбинации закона Нойберга и метода конечных элементов со критериями на базе деформация - срок службы, как бы обеспечилось точнее предусматривание срока усталости (материала) до появления инициирующего повреждения, а потом восстановление комплектного подхода для оценки срока усталости (материала). Предложенная методика использована на образцовой составной части конструкции самолета, как панель с люком, нагруженная циклическими нагрузками, как бы определился ее расчетный срок. Расчетные результаты сравнены с экспериментами. Оценка срока службы (ресурса) со применением предложенной методики находится в полном согласии с результатами экспериментов.

Ключевые слова: усталость материала, самолет, планер летательного аппарата, циклическая нагрузка, инициирующая трещина, метод конечных элементов, цифровая симуляция